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ORIGINAL RESEARCH ARTICLE

Stochastic Model to Demonstrate the Effect of Threshold Estimate Using Shock Model

V.S. Bhuvana[1] and P. Pandiyan[*2]

[1]*Department of Statistics, Manonmaniam Sundaranar University, Thirunelveli, Tamil Nadu, India*
[2]*Department of Statistics, Annamalai University, Tamil Nadu, India - 608 002*

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ABSTRACT

The threshold of the HIV plays a major role in the progression of infection. The important characteristic of the threshold for a person is the time to attain the break down point. The component fails when the total amount of damage exceeds a threshold level. The alpha-Poisson distribution is used to compute the expected time to reach the threshold status and its variance is found. The analytical results are numerically illustrated by assuming specific distributions.

Keywords: Component, Expected time, Random variable and Threshold

INTRODUCTION

One of the most urgent public-health problems in developing countries is the AIDS epidemic, caused by the HIV. The dynamic transmission of HIV is quite complex and there is no other human infection which has the same epidemiological characteristics with a similar mode of transmission. For instance, the incubation period after infection with HIV is known to be extremely long and is measured in years rather than days. In populations who choose the time of their HIV tests, independence between dates of HIV infection and HIV testing cannot be assumed.

Human immunodeficiency virus (HIV) infection is a worldwide problem and HIV/AIDS patients suffer from several opportunistic infections that occur because of poor immune system function. Poison distribution was first published in 1837. Pillai [1] and Anil [2] introduced the alpha-Poison distribution as a generalization of Poisson process. Esary, Marshall and Prochan [3] discussed that any component or device when exposed to shocks which cause damage to the device or system is likely to fail when the total accumulated damage exceeds a level called the threshold.

These assumptions are somewhat artificial, but are made because of the lack of detailed real-world information on one hand and in order to illustrate the proceedings on the other hand. Sexual contacts are the only source of HIV infection. The threshold of any individual is a random variable. If the total damage crosses a threshold level Y which itself is a random variable, the seroconversion occurs and a person is recognized as an infected. The inter-arrival times between successive contacts, the sequence of damage and the threshold are mutually independent.

Notations

X_i : a discrete random variable denoting the amount of contribution to the threshold due to the HIV transmitted in the i[th] contact, in other words the damage caused to the immune system in the i[th] contact, with p.d.f g (.) and c.d.f G (.).

Y : a discrete random variable denoting the threshold which follows alpha-Poisson distribution.

U_i : a random variable denoting the inter-arrival times between contact with c.d.f. $F_i(.)$, $i = 1,2,3 \ldots k$.

$g(.)$: The probability density functions of X_i ;

$g^*(.)$: Laplace transform of g (.)

$g_k(.)$: The k- fold convolution of g (.) i.e., p.d.f. of $\sum_{j=1}^{k} X_i$

$f(.)$: p.d.f. of random variable denoting between successive contact with the corresponding c.d.f. F (.)

$F_k(.)$: k-fold convolution of F (.); $V_k(t)$: Probability of exactly k successive contact

$S(.)$: Survival function, i.e., $P\ [T > t]$; $L(t)$: 1 - S (t)

***Corresponding Author:** P. Pandiyan , **Email:** pandiyanau@gmail.com, **Phone No:** +91- 9486648074

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RESULTS

The alpha-Poisson distribution with parameter a and α is given by

Probability generating function

$$F(x) = 1 - F\left[\alpha(1-s)^{\frac{1}{\alpha}}\right] \_\_\_(1)$$

Probability mass function

$$f(x) = \sum (-1)^k \binom{k+n}{k} \frac{(at)^{\alpha(k+n)}}{\Gamma[a(k+n)+1]}$$

When $\alpha = 1$ the survival function becomes

$$H(x) = 1 - F(x) = a - as$$

Now, $P(X_1 + X_2 + \cdots + X_k < Y)$ = P [the system does not fail, after k epochs of exits].

$$S(t) = P(X_i < Y) = \int_k^\infty g_k(x)\, H(x)\; d(x)$$

$$= [g^*(a)]^k - [g^*(as)]^k \_\_\_\_\_ (2)$$

The survival function S (t) which is the probability that an individual survives for a time t

$S(t) = P(T > t) =$ **Probability that the system survives beyond t**

$$\sum_{k=0}^{\infty} P(there\ are\ exactly\ k\ instants\ of\ exists\ in\ (0,t])^* P(the\ component\ does\ not\ fail\ in\ (0,t])$$

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t)\, P(X_i < Y)$$

$$S(t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\ [g^*(a)]^k - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\ [g^*(as)]^k \_\_\_\_\_(3)$$

Using convolution theorem for Laplace transforms, $F_0(t) = 1$ and on simplification, it can shown that

Life time $L(T) = 1 - S(t)$

Taking Laplace transform of $L(T)$, we get,

$$= 1 - \left\{\sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\ [g^*(a)]^k - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\ [g^*(as)]^k\right\}$$

$$= 1 - \left\{\left[1 - [1 - g^*(a)] \sum_{k=1}^{\infty} F_k(t)\, [g^*(a)]^{k-1}\right] - \left[1 - [1 - g^*(as)] \sum_{k=1}^{\infty} F_k(t)\, [g^*(as)]^{k-1}\right]\right\}$$

$$= 1 + [1 - g^*(a)] \sum_{k=1}^{\infty} F_k(t)\, [g^*(a)]^{k-1} + [g^*(as)] \sum_{k=1}^{\infty} F_k(t)\, [g^*(as)]^{k-1} \_\_\_\_ (4)$$

By taking Laplace-Stieltjes transform, Life time can be shown that

$$l^*(s) = 1 + \frac{[1 - g^*(a)] f^*(s)}{[1 - g^*(a) f^*(s)]} + \frac{[1 - g^*(as)] f^*(s)}{[1 - g^*(as) f^*(s)]}$$

Assuming the alpha Poisson distribution of inter-arrival times as exponential with parameter c,

$$l^*(s) = 1 + \frac{[1 - g^*(a)]}{[c + s - g^*(a)c]} + \frac{c[1 - g^*(as)]}{[c + s - g^*(as)c]} \_\_\_\_ (5)$$

Note that

$$E(T) = -\left[\frac{d}{ds} L^*(s)\right] = \frac{1}{c[1 - g^*(a)]} \_\_\_\_ (6)$$

$$E(T^2) = -\left[\frac{d^2}{ds^2} L^*(s)\right] = \frac{2}{c^2[1 - g^*(a)]^2} \_\_\_\_ (7)$$

From which $V(T) = E(T^2) - [E(T)]^2$

With the numerical value of mean and variance, we have made an attempt to find the time of leaving the service of co-variance to recruitment.

SPECIAL CASE

The distribution of total damage of threshold is Mittag-Leffler distribution with parameter a. Mittag-Leffler distributions with Laplace transform a sequence of non-negative, independently and identically distributed random variable[1] .

$$g^*(a) = \frac{1}{1 + a^\alpha} \qquad E(T) = \frac{1 + a}{c(a)} + \frac{1 + (ar)}{c(ar)} \qquad V(T) = 4\left(\frac{1 + a}{c(a)} - \frac{1 + (ar)}{c(ar)}\right)^2$$

NUMERICAL ILLUSTRATION

The theory developed was tested using stimulated data in *MathCAD* software. Developing such mathematical models, we can to some extent anticipate its spread in different populations and evaluate the potential effectiveness of different approaches for bringing the epidemic under control, and thus help to devise effective strategies to minimize the destruction caused by this epidemic

From (**Fig 1.1 & 1.2**) we could observe the difference in the values of E (T) and V (T) when the threshold distribution has alpha-Poisson distribution. If parameter value of Mittag-Leffler '*a*' increases by 0.4, 0.6 and 0.8 with fixed alpha-Poisson as α = 0.2 and the inter-arrival time 'c' increases then the expected time to recruitment and variance decreases in all the three cases. When the parameter value of Mittag-Leffler increases we find that the expected time to recruitment also decreases.

Fig 1.1: Difference in the values of E (T)

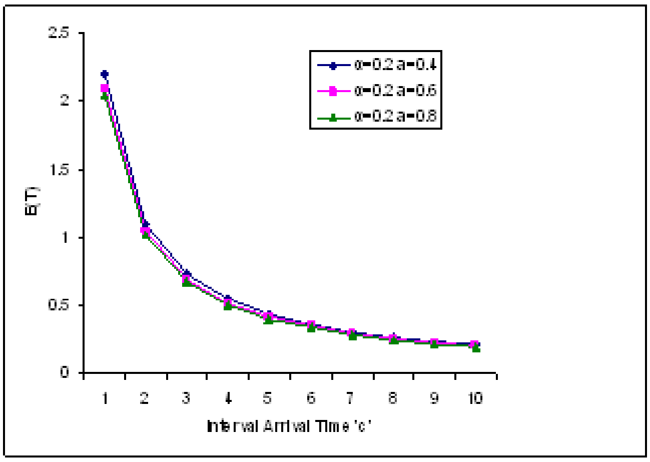
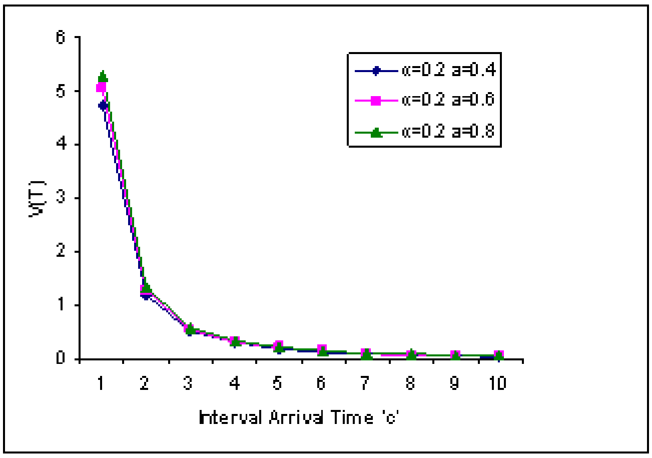


Fig 1.2: Difference in the values of V (T)



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